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Leptogenesis in unified theories with Type II see-saw

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Abstract: In some classes of flavour models based on unified theories with a type I see-saw mechanism, the prediction for the mass of the lightest right-handed neutrino is in conflict with the lower bound from the requirement of successful thermal leptogenesis. We investigate how lifting the absolute neutrino mass scale by adding a type II see-saw contribution proportional to the unit matrix can solve this problem. Generically, lifting the neutrino mass scale increases the prediction for the mass of the lightest right-handed neutrino while the decay asymmetry is enhanced and washout effects are reduced, relaxing the lower bound on the mass of the lightest right-handed neutrino from thermal leptogenesis. For instance in classes of unified theories where the lightest right-handed neutrino dominates the type I see-saw contribution, we find that thermal leptogenesis becomes possible if the neutrino mass scale is larger than about 0.15 eV, making this scenario testable by neutrinoless double beta decay experiments in the near future.

Keywords: Neutrino Physics, Baryogenesis, GUT.

Contents

1. Introduction

The mechanism of Leptogenesis [1] is one of the most attractive possibilities for explaining the observed baryon asymmetry of the universe, $n_{\mathrm{B}}/n_\gamma = (6.5^{+0.4}_{-0.8})\cdot 10^{-10}$ [2]. The asymmetry is generated via the out-of-equilibrium decay of the same heavy right-handed neutrinos which are responsible for generating naturally small neutrino masses in the type I see-saw scenario [3]. The thermal version of leptogenesis in the so-called strong washout regime is thereby virtually independent of initial conditions, since the effect of any pre-existing baryon asymmetry or right-handed neutrino abundance is washed out by processes in the thermal bath involving the lightest right-handed neutrino. In the type I see-saw mechanism, thermal leptogenesis (assuming hierarchical right-handed neutrino masses) puts strong constraints on the parameters of the see-saw mechanism. To start with, the decay asymmetries are bounded from above and best asymmetry is achieved for hierarchical neutrino masses [4, 5]. This leads to a lower bound on the masses of the right-handed neutrinos [5], which amounts about 10^9 GeV (see e.g. [6, 7] for recent calculations) for hierarchical neutrino masses and increases strongly as the neutrino mass scale increases. Together with the observation that in the type I see-saw scenario, the washout parameter $\widetilde{m}_1$ [8] increases with increasing neutrino mass scale leading to strongly enhanced washout which makes leptogenesis less efficient, this enables a bound on the absolute neutrino mass scale of about 0.1 eV to be derived [9].

In models with a left-right symmetric particle content like minimal left-right symmetric models, Pati-Salam models or Grand Unified Theories (GUTs) based on SO(10), the type I see-saw mechanism is typically generalized to a type II see-saw [10], where an additional direct mass term $m^{\mathrm{II}}_{\mathrm{LL}}$ for the light neutrinos is present. The effective mass matrix of the light neutrinos is then given by

$$m^{\nu}_{\mathrm{LL}} = m^{\mathrm{I}}_{\mathrm{LL}} + m^{\mathrm{II}}_{\mathrm{LL}}\ , \quad \text{where} \quad m^{\mathrm{I}}_{\mathrm{LL}} = -v_{\mathrm{u}}^2\, Y_\nu\, M^{-1}_{\mathrm{RR}}\, Y_\nu^T \tag{1.1}$$

is the type I see-saw mass matrix with Y_ν being the neutrino Yukawa matrix in left-right convention, M_{RR} the mass matrix of the right-handed neutrinos and $v_{\mathrm{u}} = \langle h^0_{\mathrm{u}} \rangle$ is the vacuum expectation value (vev) which leads to masses for the up-type quarks. From a rather model independent viewpoint, the type II mass term can be considered as an additional contribution to the lowest dimensional effective neutrino mass operator. In most explicit models, the type II contribution stems from a see-saw suppressed induced vev of a $\mathrm{SU}(2)_{\mathrm{L}}$-triplet Higgs field.

Leptogenesis in type II see-saw scenarios [11 − 13] via the decay of the lightest right-handed neutrino provides a natural generalization of type I leptogenesis. In the limit that the mass of the lightest right-handed neutrino is much lighter than the other particles participating in the see-saw mechanism, the decay asymmetry depends just on the low energy neutrino mass matrix $m^{\nu}_{\mathrm{LL}} = m^{\mathrm{I}}_{\mathrm{LL}} + m^{\mathrm{II}}_{\mathrm{LL}}$ and on the Yukawa couplings to the lightest right-handed neutrino and its mass [12]. It has been shown that type II leptogenesis puts constraints on the see-saw parameters as well, which however differ substantially from the constraints in the type I case. For instance, the bound on the decay asymmetry increases with increasing neutrino mass scale [12], in contrast to the type I case where it decreases. As a consequence, the lower bound on the mass of the lightest right-handed neutrino from leptogenesis decreases for increasing neutrino mass scale [12]. Finally, since the type II contribution typically does not affect washout, there is no bound on the absolute neutrino mass scale in type II leptogenesis [11].

One potential problem for thermal leptogenesis emerges in some classes of unified theories, where the neutrino Yukawa couplings are linked to the Yukawa couplings in the up-quark sector which implies small Dirac mixing from the Yukawa matrices [14, 15]. In such models the masses of the right-handed neutrinos calculated within the type I see-saw mechanism are required to be strongly hierarchical, and the lightest right-handed neutrino turns out to be so light that it can be below the leptogenesis bound of 10^9 GeV. Within the type I see-saw mechanism, proposed solutions to this potential problem include nearly degenerate right-handed neutrinos leading to resonant leptogenesis [16 − 18], non-thermal leptogenesis via the decay of the inflaton [19, 20], or of course applying a completely different baryogenesis mechanism. The fact that the type II see-saw mechanism has the potential for solving the right-handed neutrino problems in unified theories was also mentioned in [15], but not discussed in any detail.

In this paper we consider realistic classes of unified theories [21, 22], where the lightest right-handed neutrino dominates the type I see-saw mechanism [23]. We show that in this scenario, the prediction for the mass of the lightest right-handed neutrino is generically in conflict with the lower bound from the requirement of successful thermal leptogenesis.

Although our predictions for the masses of the right-handed neutrinos are somewhat larger than the estimated range given in [15], we show that in such models leptogenesis is strongly washed out, leading to a more stringent lower limit on the right-handed neutrino mass of about 10^{11} GeV which is in conflict with the allowed range of right-handed neutrino masses from the class of unified model considered.

The main purpose of this paper is to investigate how lifting the absolute neutrino mass scale by adding a type II see-saw contribution proportional to the unit matrix can lead to a resolution of the conflict between the leptogenesis lower bound on the lightest right-handed neutrino mass, and the allowed range of lightest right-handed neutrino masses in classes of unified theories where the neutrino Yukawa couplings are related to the up-quark ones. We have previously shown that such a "type II upgrade" [24] provides a natural way for transforming a type I see-saw model for hierarchical neutrino masses into a type II see-saw model for quasi-degenerate neutrinos [25]. Increasing the neutrino mass scale using the type II see-saw mechanism implies that the mass splittings between the physical neutrino masses are reduced, and since in this approach these splittings are controlled by the type I see-saw mechanism, this has the effect of increasing the masses of the right-handed neutrinos required to give a successful description of neutrino masses. Increasing the type II contributions also implies that the decay asymmetries become larger and washout effects are reduced, which reduces the lower bound on the mass of the lightest right-handed neutrino from thermal leptogenesis. The combination of these two effects implies that, as the type II neutrino mass scale increases, the increasing lightest right-handed neutrino mass prediction converges with the decreasing leptogenesis lower limit, thereby resolving the conflict between unified theories and thermal leptogenesis. Quantitatively we find that the conflict is resolved for a neutrino mass scale larger than about 0.15 eV. Our scheme therefore predicts a signal in neutrinoless double beta decay experiments (and possibly also in direct searches for neutrino mass) [26, 27] in the near future. An additional nice feature of our proposal is that for such neutrino masses, thermal leptogenesis remains in the so-called strong washout regime, where the produced baryon asymmetry is virtually independent of initial conditions.

It is worth mentioning that an analogous problem appears in unified theories where the lightest right-handed neutrino determines the sub-dominant contributions to the neutrino mass matrix, and thermal leptogenesis requires a similar lift of the neutrino mass scale in this case. On the contrary, the above-mentioned conflict is typically absent, if the heaviest right-handed neutrino is dominant [23, 28] and it can be ameliorated if the dominance conditions are relaxed [29].

2. Leptogenesis in unified theories with Type I see-saw

2.1 Unified models with dominant lightest RH neutrino ν_{R1}

In order to discuss predictions for right-handed neutrino masses and issues of thermal leptogenesis explicitly, it is necessary to make assumptions. We will therefore consider first a class of unified models motivated by left-right symmetric unified theories such as

GUTs based on SO(10), where the lightest right-handed neutrino $\nu_{\rm R1}$ dominates the see-saw mechanism. In these classes of models we are led to specific forms of the Yukawa couplings, which we will now briefly review. More details and explicit examples for models within this class of unified flavour models can be found in ref. [21, 22].

The known experimental data about fermion masses and mixings can be successfully accommodated by Yukawa matrices for up-type quarks $Y_{\rm u}$, down-type quarks $Y_{\rm e}$, charged leptons $Y_{\rm d}$ and neutrinos Y_ν all being of the form

$$Y_f \sim \begin{pmatrix} 0 & \epsilon_f^3 & \epsilon_f^3 \\ \epsilon_f^3 & x_f \epsilon_f^2 + \epsilon_f^3 & x_f \epsilon_f^2 + \epsilon_f^3 \\ \epsilon_f^3 & x_f \epsilon_f^2 + \epsilon_f^3 & \mathcal{O}(1) \end{pmatrix}, \tag{2.1}$$

where $f = {\rm u}, {\rm d}, {\rm e}, \nu$ and where in the quark sector $\epsilon_{\rm u} \approx 0.05$ and $\epsilon_{\rm d} \approx 0.15$ are different expansion parameters obtained from a fit to $\frac{m_{\rm c}}{m_{\rm t}}$ and $\frac{m_{\rm s}}{m_{\rm b}}$. In the charged lepton sector, a Clebsch factor x_f (Georgi-Jarlskog factor [30]) of $x_e = -3$ in the (2,2)-entry of $Y_{\rm e}$ typically arises and the expansion parameter $\epsilon_{\rm e}$ is equal to $\epsilon_{\rm d}$. For example the Clebsch factor x_f can be proportional to weak hypercharge [21, 22], suggesting $x_{\rm d} = -1, x_{\rm u} = 2, x_{\rm e} = -3$ and $x_\nu = 0$. The approximate texture zero in the (0,0)-entry of Y_f furthermore leads to the successful GST relation [31], which relates quark masses and the Cabibbo angle. Note that the Yukawa matrices in eq. (2.1) are written in a left-right convention in which the first column gives the couplings to the first right-handed fermion, and so on.

Although not unique, the texture in eq. (2.1) has the feature that all the charged fermion mixing angles are small, which is common to many SO(10) type models. Within this class of models, we shall obtain large neutrino mixing using a mechanism called light sequential dominance (LSD), in which the lightest right-handed neutrino dominates the type I see-saw contribution to the atmospheric mass, and the next-to-lightest right-handed neutrino dominates the type I see-saw contribution to the solar neutrino mass [23]. Although the choice of LSD is also not unique, it has the desirable feature that a neutrino mass hierarchy arises naturally without any tuning, since the large neutrino mixing angles are given by ratios of Yukawa couplings, and the problem of large neutrino mixing is therefore decoupled from the neutrino mass hierarchy which arises naturally from the sequential dominance of the three right-handed neutrinos.

A concrete example of the above class of models was recently given in ref. [22], where a model based on SO(3) family symmetry and Pati-Salam unification was presented. A systematic operator expansion for the operators responsible for Yukawa and Majorana matrices, where an additional flavour symmetry was introduced to control the operator expansion, was performed. The resulting Yukawa matrices resembled those given in eq. (2.1), where the Clebsch factors x_f are proportional to the hypercharge generator of the right-handed fermions. The Majorana mass matrix for the right-handed neutrinos was determined to be strongly hierarchical, leading to large (approximate tri-bimaximal) neutrino mixing via LSD. The hierarchical nature of the Yukawa and Majorana mass matrix follows as a consequence of the additional flavour symmetry, and, since the operator expansion is controlled by a flavour symmetry, the high powers of expansion parameter appearing are technically natural in the sense that if a small parameter is set to zero the symmetry is enlarged. The model is also technically natural in the sense that the low energy predictions are insensitive to the choice of high energy input parameters (a general feature of sequential dominance). The existence of such a model underpins the theoretical approach followed in this paper, and for example justifies the choice of neutrino Yukawa matrix and Majorana masses assumed in the next sub-section. Since the left-handed Pati-Salam matter fields of the model form SO(3)-triplets, it is also possible in principle to "up-grade" this model to include a type II see-saw contribution to the neutrino masses, along the lines of the approach discussed in section 3.

2.2 Estimating the lightest RH neutrino mass M_{R1} from unification

In the classes of models outlined above (e.g. the model in [22]) the neutrino Yukawa matrix has the form

$$Y_\nu = \begin{pmatrix} 0 & a\epsilon^3 & p\epsilon^3 \\ e\epsilon^3 & b\epsilon^3 & q\epsilon^3 \\ f\epsilon^3 & c\epsilon^3 & \mathcal{O}(1) \end{pmatrix} , \tag{2.2}$$

where a, b, c, e, f, p, q are order unity dimensionless couplings, and $\epsilon := \epsilon_\nu = \epsilon_{\mathrm{u}}$. Note that the entries proportional to ϵ^2 are absent due to a vanishing Clebsch factor $x_\nu = 0$. Providing the lightest right-handed neutrino (corresponding to the first column in eq. (2.2)) provides the dominant type I see-saw contribution to the atmospheric neutrino mass, then we are naturally led to large atmospheric neutrino mixing $\tan\theta_{23} \approx e/f \sim 1$ for $e \sim f$ and a hierarchical neutrino mass spectrum, even with an approximately diagonal mass matrix

$$M_{\mathrm{RR}} \ = \ \mathrm{diag}\,(M_{\mathrm{R1}}, M_{\mathrm{R2}}, M_{\mathrm{R3}}) \tag{2.3}$$

for the right-handed neutrinos. This is the single right-handed neutrino dominance mechanism [23]. Furthermore, if the next-to-lightest right-handed neutrino (corresponding to the second column in eq. (2.2)) provides the dominant type I see-saw contribution to the solar neutrino mass, then we are naturally led to large solar mixing $\tan\theta_{12} \approx \sqrt{2}a/(b-c)$ for $a \sim b \sim c$, which is the sequential neutrino dominance mechanism [23]. In the following, we will assume the sequential dominance conditions [23]

$$\frac{|e|^2\epsilon^6, |f|^2\epsilon^6}{M_{\mathrm{R1}}} \gg \frac{|a|^2\epsilon^6, |b|^2\epsilon^6, |c|^2\epsilon^6}{M_{\mathrm{R2}}} \gg \frac{1}{M_{\mathrm{R3}}} , \tag{2.4}$$

which immediately leads to a physical neutrino mass hierarchy $m_1 \ll m_2 \ll m_3$. It also implies a hierarchy of heavy right-handed neutrino masses $M_{\mathrm{R1}} \ll M_{\mathrm{R2}} \ll M_{\mathrm{R3}}$. The mass of the lightest right-handed neutrino, which dominates the type I see-saw mechanism, is then given by[1] [23]

$$M_{\mathrm{R1}} \ = \ \frac{(e\epsilon^3)^2 v_{\mathrm{u}}^2}{(s_{23}^\nu)^2 m_3^{\mathrm{I}}} , \tag{2.5}$$

where the contribution to the masses of the light neutrinos from the type I see-saw mechanism are denoted by m_i^{I}, $i = 1, 2, 3$, in order to distinguish them from the type II

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[1] For example a lightest right-handed neutrino of this mass was naturally obtained in [22].

see-saw contribution we will introduce in section 3. In the type I see-saw case with a hierarchical neutrino mass spectrum, m_3^{I} is simply given by $\sqrt{|\Delta m^2_{\mathrm{atm}}|}$ with $|\Delta m^2_{\mathrm{atm}}| \approx 2.2 \cdot 10^{-3}\,\mathrm{eV}^2$ [32]. Nearly maximal mixing θ_{23} stems from the neutrino sector, i.e. $s_{23} \approx s^{\nu}_{23}$ with only small corrections from the charged leptons, and we use $s^{\nu}_{23} = 1/\sqrt{2}$ in the following. Furthermore, in the classes of models outlined above, the neutrino Yukawa matrix Y_ν is related to the up-type quark Yukawa matrix Y_{u}, although it is of course not required that both Yukawa matrices have to be identical. For estimating M_{R1}, we are interested in the (2,1)-entry of Y_ν which is equal to $e\epsilon^3$. Explicit fits of the quark sector suggest that $(Y_{\mathrm{u}})_{12} = (Y_{\mathrm{u}})_{21} = 1.5\epsilon_{\mathrm{u}}^3$ [33] at M_{GUT}, and for our estimates we will allow $(Y_\nu)_{21}$ to vary from about $\frac{1}{5} \cdot (Y_{\mathrm{u}})_{21}$ to $5 \cdot (Y_{\mathrm{u}})_{21}$. Such differences from the quark Yukawa matrix might stem from different Clebsch factors and/or from uncertainties in the quark masses which lead to uncertainties in ϵ. With $e\epsilon^3 \in [\frac{1}{5}, 5] \cdot 1.5\epsilon^3$, we obtain

$$M_{\mathrm{R1}} \;\sim\; 2 \cdot 10^6\;\mathrm{GeV} \;\cdots\; 1 \cdot 10^9\;\mathrm{GeV}\;, \tag{2.6}$$

neglecting RG corrections at this stage. The range for M_{R1} can of course be extended/reduced somewhat by assuming a larger/smaller range for $e\epsilon^3$. Note that although it seems that the range of eq. (2.6) is marginally consistent with the absolute lower bound on M_{R1} of about 10^9 GeV [5] from thermal leptogenesis, we will show below that for the lightest right-handed neutrino dominating the see-saw mechanism (LSD), this bound is in fact much more stringent and in gross conflict with the predicted range for M_{R1} of eq. (2.6).

2.3 Lower bound on M_{R1} from thermal leptogenesis

The observed baryon asymmetry of the universe is given by $n_{\mathrm{B}}/n_\gamma = (6.5^{+0.4}_{-0.8}) \cdot 10^{-10}$ [2]. This has to be compared to the baryon-to-photon ratio produced by leptogenesis which can be calculated from the formula (using a notation as, e.g., in [6])

$$\frac{n_{\mathrm{B}}}{n_\gamma} \approx -\,1.04 \cdot 10^{-2}\,\varepsilon_1\,\eta\;, \tag{2.7}$$

where ε_1 is the decay asymmetry of the lightest right-handed neutrino into lepton doublet and Higgs and where the parameter η is the so-called efficiency factor, which e.g. takes dilution of the produced asymmetry by washout processes into account.

For the type I see-saw mechanism, the decay asymmetry ε_1 [34] in the MSSM can be written as

$$\begin{aligned} \varepsilon_1 &= \frac{1}{8\pi} \frac{\sum_{j\neq 1} \mathrm{Im}\,[(Y_\nu^\dagger Y_\nu)^2_{1j}]}{\sum_f |(Y_\nu)_{f1}|^2} \sqrt{x_j} \left[\frac{2}{1-x_j} - \ln\left(\frac{x_j+1}{x_j} \right) \right] \\ &\approx \frac{-3}{8\pi} \frac{\mathrm{Im}\,[(Y_\nu^\dagger Y_\nu)^2_{12}]}{\sum_i |(Y_\nu)_{i1}|^2} \frac{M_{\mathrm{R1}}}{M_{\mathrm{R2}}} = \frac{3}{8\pi} \frac{M_{\mathrm{R1}}}{v_{\mathrm{u}}^2} \frac{\sum_{fg} \mathrm{Im}\,[(Y_\nu^*)_{f1} (Y_\nu^*)_{g1} (m^{\mathrm{I}}_{\mathrm{LL}})_{fg}]}{(Y_\nu^\dagger Y_\nu)_{11}} \end{aligned} \tag{2.8}$$

with $x_j := M^2_{\mathrm{R}j}/M^2_{\mathrm{R1}}$ for $j \neq 1$. In the second line, we have used that M_{R3} effectively decouples from the see-saw mechanism and from leptogenesis and that $M_{\mathrm{R1}} \ll M_{\mathrm{R2}} \ll M_{\mathrm{R3}}$ (cf. eq. (2.4)).

The efficiency factor η can be computed from a set of coupled Boltzmann equations (see e.g. [8]) and it is subject to e.g. thermal correction [6] and corrections from spectator processes [35], $\Delta\mathrm{L}=1$ processes involving gauge bosons [17, 6] and from renormalization group running [36]. For $M_{\mathrm{R}1}$ much smaller than 10^{14} GeV [6], to a good approximation the efficiency factor depends only on the quantity $\widetilde{m}_1$ [8], defined by

$$\widetilde{m}_1 := \frac{\sum_f (Y_\nu^\dagger)_{1f}(Y_\nu)_{f1}\, v_\mathrm{u}^2}{M_{\mathrm{R}1}} \,. \tag{2.9}$$

For $\widetilde{m}_1$ larger than about 10^{-2} eV, η is independent of the initial population of right-handed (s)neutrinos (see e.g. figure 8 of [6]). In this range, larger $\widetilde{m}_1$ means larger washout and a reduced efficiency factor η. For η, we will use the results provided by the authors of [6], i.e. a numerical fit to a large set of numerical results for η in the MSSM for different values of $\widetilde{m}_1$ and $M_{\mathrm{R}1}$.

In the type I see-saw mechanism, there is a bound on the decay asymmetry, which amounts to [4, 5]

$$|\varepsilon_1| \leq \frac{3M_{\mathrm{R}1}}{8\pi v_\mathrm{u}^2}(m_3^\mathrm{I} - m_1^\mathrm{I}) \leq \frac{3M_{\mathrm{R}1}}{8\pi v_\mathrm{u}^2}\sqrt{|\Delta m^2_{\mathrm{atm}}|} \tag{2.10}$$

in the MSSM. This leads to a lower bound on the mass of the lightest right-handed neutrino [5]. Assuming best efficiency, i.e. $\widetilde{m}_1$ around 10^{-3} eV for zero initial population of $\nu_{\mathrm{R}1}$, the bound is about $M_{\mathrm{R}1} \geq 10^9$ GeV. As we will see below, the realistic bound in the considered classes of unified models is much higher. Furthermore, the bound on $M_{\mathrm{R}1}$ increases for increasing absolute neutrino mass scale. This is because thermal type I leptogenesis is less efficient for a larger neutrino mass scale since [7]

$$\widetilde{m}_1 \geq m^\mathrm{I}_{i,\mathrm{min}} \,, \tag{2.11}$$

with $m^\mathrm{I}_{i,\mathrm{min}} := \min(m_1^\mathrm{I}, m_2^\mathrm{I}, m_3^\mathrm{I})$. Together with an improved bound on the type I decay asymmetry, this finally leads to an upper bound for the absolute mass scale of the light neutrinos of about 0.1 eV [9]. Let us note at this point that if neutrinoless double beta decay or a signal for neutrino mass from direct searches is observed in the near future and would point to a mass above 0.1 eV, the requirement of successful thermal leptogenesis would disfavour the type I see-saw mechanism, strongly pointing towards a type II see-saw.

In the case that the lightest right-handed neutrino dominates the see-saw mechanism, ε_1 is typically proportional to $m_2^\mathrm{I} = \sqrt{\Delta m^2_{\mathrm{sol}}}$, a factor of $\sqrt{m^2_{\mathrm{sol}}/|m^2_{\mathrm{atm}}|}$ smaller than the upper bound in eq. (2.10) [28]. In our analysis, we will however use the general bound of eq. (2.10). With respect to the parameter $\widetilde{m}_1$ which governs washout of the produced asymmetry, from eqs. (2.2) and (2.9) we now obtain [28]

$$\widetilde{m}_1 = m_3^\mathrm{I} \,. \tag{2.12}$$

This implies large washout effects compared to its optimal value for $\widetilde{m}_1 \approx 10^{-3}$ eV and the efficiency factor η is significantly reduced. Using the results for η from [6],[2] the bound

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[2] Note that there are minor differences between the results quoted in the literature. However due to the large uncertainties we allow for our estimates this differences are not significant for our analysis.

on M_{R1} for a dominant lightest right-handed neutrino can be calculated from (combining eqs. (2.7), (2.10) and (2.12))

$$M_{\mathrm{R1}} \geq \frac{8\pi v_{\mathrm{u}}^2}{3} \frac{n_{\mathrm{B}}/n_\gamma}{1.04 \cdot 10^{-2} \sqrt{|\Delta m^2_{\mathrm{atm}}|}} \frac{1}{\eta} \gtrsim 10^{11}\ \mathrm{GeV}\ , \tag{2.13}$$

where we have used the present best-fit values $\Delta m^2_{\mathrm{atm}} = 2.2 \cdot 10^{-3}\,\mathrm{eV}^2$ [32] and $n_{\mathrm{B}}/n_\gamma = 6.5 \cdot 10^{-10}$ [2] and where η in eq. (2.13) is calculated with $\widetilde{m}_1 = m_3^{\mathrm{I}} = \sqrt{\Delta m^2_{\mathrm{atm}}}$, yielding $\eta \approx 0.003$ [6]. A similar conclusion has been obtained in [37], where leptogenesis with two right-handed neutrinos and a texture zero in the (0,0)-entry of Y_ν has been analyzed.

The bound of eq. (2.13) is clearly in conflict with the range $M_{\mathrm{R1}} \sim 2 \cdot 10^6\ \mathrm{GeV}\ \cdots\ 1 \cdot 10^9\,\mathrm{GeV}$ (see eq. (2.6)) estimated for the class of unified models discussed above. As briefly discussed above, proposed solutions to this potential problem within the framework of the leptogenesis mechanism might make use of non-thermal leptogenesis via the decay of the inflaton [19]. On the other hand, resonant leptogenesis [16] does not seem to appear natural in the considered scenario, but might well be applied to other classes of unified flavour models [18]. Here our preferred route towards resolving the above conflict is to generalize the type I see-saw mechanism to a type II see-saw. In the following section, we will show how raising the absolute neutrino mass scale by adding a type II see-saw contribution proportional to the unit matrix can resolve the conflict between the leptogenesis bound and the prediction for M_{R1}.

3. Leptogenesis in unified theories with Type II see-saw

Extending the type I see-saw [3] to a type II see-saw mechanism [10] by an additional direct mass term for the light left-handed neutrinos has interesting consequences for leptogenesis [11 − 13]. The type II see-saw mechanism also opens up new possibilities for constructing models of fermion masses and mixings. We have previously shown [24] how adding a type II contribution proportional to the unit matrix to the neutrino mass matrix, $m_{\mathrm{LL}} = -v_{\mathrm{u}}^2 Y_\nu M_{\mathrm{RR}}^{-1} Y_\nu^T + m^{\mathrm{II}} e^{\mathrm{i}\delta_\Delta} \mathbb{1}$, allows models with hierarchical neutrino masses to be transformed into type II see-saw models with a partially degenerate mass spectrum in a natural way. Schematically, the structure of the neutrino mass matrix is given by

$$m^\nu_{\mathrm{LL}} \approx m^{\mathrm{II}} e^{\mathrm{i}\delta_\Delta} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \begin{pmatrix} (m^{\mathrm{I}}_{\mathrm{LL}})_{11} & (m^{\mathrm{I}}_{\mathrm{LL}})_{12} & (m^{\mathrm{I}}_{\mathrm{LL}})_{13} \\ (m^{\mathrm{I}}_{\mathrm{LL}})_{21} & (m^{\mathrm{I}}_{\mathrm{LL}})_{22} & (m^{\mathrm{I}}_{\mathrm{LL}})_{23} \\ (m^{\mathrm{I}}_{\mathrm{LL}})_{31} & (m^{\mathrm{I}}_{\mathrm{LL}})_{32} & (m^{\mathrm{I}}_{\mathrm{LL}})_{33} \end{pmatrix} . \tag{3.1}$$

Such a “type II upgrade” of hierarchical type I see-saw models has been analyzed systematically in [24] and classes of type II see-saw models have been proposed which use SO(3) flavour symmetry and a real vacuum alignment. The type II part proportional to the unit matrix governs the neutrino mass scale, whereas the hierarchical type I part controls the neutrino mass splittings and the mixing angles, e.g. using sequential dominance [23] within the type I see-saw contribution. Although issues of unification are not addressed in [24], the general strategy may well be applied to unified theories. For example, a Pati-Salam unified model with type I see-saw based on SO(3) flavour symmetry and a real vacuum alignment has recently been proposed [22], which has a neutrino Yukawa matrix of the class outlined in section 2.1 and which can in principle be "up-graded" to include a II see-saw contribution. We will now discuss how lifting the absolute neutrino mass scale by adding a type II see-saw contribution $m^{\mathrm{II}}\mathbb{1}$ can solve the potential conflict between the prediction for $M_{\mathrm{R}1}$ and the lower bound from leptogenesis. As in the previous section, we will focus on classes of unified flavour models where the lightest right-handed neutrino dominates the type I see-saw contribution as an example. In the next sub-section we shall show that the lightest right-handed neutrino mass increases with increasing type II neutrino mass scale. Then in the following sub-section we shall show that the thermal leptogenesis lower bound on the lightest right-handed neutrino mass decreases with increasing type II mass scale. The combination of these two effects then resolves the right-handed neutrino mass conflict between unified theories and thermal leptogenesis for a sufficiently high type II neutrino mass scale, which we shall subsequently estimate.

3.1 Right-handed neutrino masses and Type II see-saw

Following section 2.1, where the classes of unified type I see-saw models are discussed, we consider the neutrino Yukawa matrix of the form

$$Y_\nu = \begin{pmatrix} 0 & a\epsilon^3 e^{\mathrm{i}\delta_2} & * \\ e\epsilon^3 e^{\mathrm{i}\delta_1} & b\epsilon^3 e^{\mathrm{i}\delta_2} & * \\ f\epsilon^3 e^{\mathrm{i}\delta_1} & c\epsilon^3 e^{\mathrm{i}\delta_2} & \mathcal{O}(1) \end{pmatrix} . \tag{3.2}$$

The specific phase structure arises from the real alignment of the SO(3) breaking vacuum as in [38, 24, 39, 22], which we have chosen for definiteness. Compared to eq. (2.2), e, f, q, b, c are now real (not necessarily positive) parameters and δ_1, δ_2 are common phases for each column of Y_ν.

For example a Pati-Salam unified model based on SO(3) with a neutrino Yukawa matrix similar to eq. (3.2) has recently been proposed [22]. In the proposed model a vacuum alignment with $a = b = c$ and $e = -f$ is used to give tri-bimaximal neutrino mixing, but results in zero type I leptogenesis [22]. Such a model may in principle be "up-graded" to a type II model along the lines discussed here, allowing successful leptogenesis. This provides a good example of the type of model to which the results presented here may be applied. However, models based on SU(3) [21] cannot similarly be "up-graded".

Note that in realistic models the phase structure in eq. (3.2) may be modified by correction from higher-dimensional, next-to-leading operators. The entries marked with a star are much smaller than 1 and do not play any role in our analysis. We will furthermore make use of the fact that in the considered class of models, only small corrections to the neutrino mixings, compared to the present experimental uncertainties, arise from the charged lepton sector. We will neglect these corrections in the following since they only contribute marginally to the uncertainties for the estimates of $M_{\mathrm{R}1}$ and do not effect the leptogenesis bounds.

Using the sequential dominance conditions in eq. (2.4) for the type I contribution to the neutrino mass matrix and approximating $m_1^{\mathrm{I}} = 0$, the total masses of the light neutrinos, the eigenvalues of $m^\nu_{LL} = m^{\rm I}_{LL} + m^{\rm II}_{LL}$, are given by

$$m_1 \approx m^{\rm II} \, , \tag{3.3a}$$

$$m_2 \approx |m^{\rm II} e^{{\rm i}\delta_\Delta} - m^{\rm I}_2 \, e^{i2\delta_2}| \, , \tag{3.3b}$$

$$m_3 \approx |m^{\rm II} e^{{\rm i}\delta_\Delta} - m^{\rm I}_3 \, e^{i2\delta_1}| \, , \tag{3.3c}$$

where $\{0, m^{\rm I}_2, m^{\rm I}_3\}$ are the approximate mass eigenvalues of the type I contribution to the neutrino mass matrix, $m^{\rm I}_{\rm LL}$, and $m^{\rm II}$ is defined to be positive.

We can now calculate analytically how the mass of $M_{\rm R1}$ depends on $m^{\rm II}$, which is equal to the mass of the lightest left-handed neutrino for a normal mass ordering. Let us therefore first extract $m^{\rm I}_3$. Clearly, since $m^{\rm I}_3$ generates the mass splitting of m_3 and m_1, for given $|\Delta m^2_{\rm atm}| := |m_3^2 - m_1^2|$ it has to decrease if the absolute neutrino mass scale is lifted via $m^{\rm II}$. From eqs. (3.3c), we obtain

$$m^{\rm I}_3 = m^{\rm II} \cos(2\delta_1 - \delta_\Delta) \pm \sqrt{[m^{\rm II} \cos(2\delta_1 - \delta_\Delta)]^2 \pm |\Delta m^2_{\rm atm}|} \, , \tag{3.4}$$

where the '+' stands for normal ordering of the mass eigenvalues, i.e. $\cos(2\delta_1 - \delta_\Delta) < 0$, and the '$-$' stands for an inverse ordering corresponding to $\cos(2\delta_1 - \delta_\Delta) > 0$ (if a solution exists which is obviously not guaranteed in the latter case for small $m^{\rm II}$). A graphical illustration can be found in figure 3 of ref. [24]. Assuming a normal mass ordering, for $[m^{\rm II} \cos(2\delta_1 - \delta_\Delta)]^2 \gg |\Delta m^2_{\rm atm}|$, we obtain

$$m^{\rm I}_3 \sim \frac{\Delta m^2_{\rm atm}}{-2 m^{\rm II} \cos(2\delta_1 - \delta_\Delta)} \, , \tag{3.5}$$

which shows that the type I mass contributions (which govern the neutrino mass splittings in this approach) decrease with increasing type II neutrino mass scale. Finally, from $m^{\rm I}_3$, $M_{\rm R1}$ is given by

$$M_{\rm R1} = \frac{(e\epsilon^3)^2 v_{\rm u}^2}{s^2_{23} m^{\rm I}_3} \, , \tag{3.6}$$

analogous to eq. (2.5). However, compared to the type I case, $m^{\rm I}_3$ can now be significantly smaller than $\sqrt{|\Delta m^2_{\rm atm}|}$ for $m^{\rm II}$ close to the present bounds for the absolute neutrino mass scale. Thus the prediction for $M_{\rm R1}$ increases with increasing type II neutrino mass scale, as claimed earlier. We estimate that, for $m^{\rm II} = 0.2\,$eV, $m^{\rm I}_3$ is reduced from about 0.05 eV to 0.006 eV and thus the prediction for $M_{\rm R1}$ increases by about an order of magnitude.

3.2 Leptogenesis bound on $M_{\rm R1}$ and Type II see-saw

In the class of models under consideration, if the lightest right-handed neutrino dominates the type I see-saw contribution to the neutrino mass matrix, thermal leptogenesis becomes more efficient when the type II neutrino mass scale increases in two ways: due to an enhanced decay asymmetry and due to reduced washout leading to a larger efficiency factor η. Let us now discuss these points in detail. They both result in a decrease of the lower bound on $M_{\rm R1}$ from thermal leptogenesis.

The decay asymmetry in the type II see-saw, which generalizes the type I decay asymmetry of eq. (2.8), is given by [11, 12]

$$\varepsilon_1 = \frac{3}{8\pi}\frac{M_{\mathrm{R1}}}{v_{\mathrm{u}}^2}\frac{\sum_{fg}\mathrm{Im}\left[(Y_\nu^*)_{f1}(Y_\nu^*)_{g1}(m_{\mathrm{LL}}^{\mathrm{I}}+m_{\mathrm{LL}}^{\mathrm{II}})_{fg}\right]}{(Y_\nu^\dagger Y_\nu)_{11}} \tag{3.7}$$

in the limit that the lightest right-handed neutrino is much lighter than the additional particles associated with the type I and type II see-saw mechanism (e.g. much lighter than the $\mathrm{SU}(2)_{\mathrm{L}}$-triplet Higgs fields). It is bounded from above by [12]

$$|\varepsilon_1| \leq \frac{3M_{\mathrm{R1}}}{8\pi v_{\mathrm{u}}^2} m_{i,\max}^{\nu}\,, \tag{3.8}$$

with $m_{i,\max}^{\nu} := \max(m_1, m_2, m_3)$. Type I and type II bounds on ε_1 are identical for a hierarchical neutrino mass spectrum [11, 12]. However, if the neutrino mass scale m^{II} increases, the type II bound increases [12] whereas the type I bound decreases [5].

Explicitly, if we add a type II see-saw contribution proportional to the unit matrix to the class of type I see-saw models under consideration, we obtain

$$\varepsilon_1 = -\frac{3M_{\mathrm{R1}}}{8\pi v_{\mathrm{u}}^2}\left[\sin(2\delta_1 - \delta_\Delta)m^{\mathrm{II}} \pm \mathcal{O}(m_2^{\mathrm{I}})\right]. \tag{3.9}$$

When increasing the absolute neutrino mass scale, m_2^{I} decreases very fast (see e.g. figure 5(a) of ref. [24]) and the decay asymmetry is typically dominated by the type II contribution already for m^{II} larger than about 0.03 eV. We note that the bound on ε_1 of eq. (3.8) can be nearly saturated with a type II see-saw contribution proportional to the unit matrix in a natural way. If the type II contribution to the decay asymmetry dominates leptogenesis, the “leptogenesis phase” in our scenario is given by

$$\delta_{\mathrm{cosm}} = 2\delta_1 - \delta_\Delta\,. \tag{3.10}$$

The decay asymmetry dominantly stems from the interference of the tree-level decay of ν_{R1} with the one-loop diagram where the triplet responsible for the type II see-saw contribution or its superpartner run in the loop (see figure 1). It is interesting to note that although classes of “type-II-upgraded” see-saw models studied in [24] have the generic property that all low energy observable CP phases from the neutrino sector become smaller as the neutrino mass scale increases (e.g. the Dirac CP phase δ observable in neutrino oscillations), the phase δ_{cosm} relevant for leptogenesis is unaffected and remains finite in the large type II mass limit.

In type II leptogenesis with M_{R1} much lighter than other contributions to the see-saw mechanism, the efficiency factor η is typically still determined by M_{R1} and the Yukawa couplings to ν_{R1} [11] and, in particular, washout effects from $\Delta L=2$-scattering processes involving the $\mathrm{SU}(2)_{\mathrm{L}}$-triplets are negligible for $M_\Delta \gg M_{\mathrm{R1}}$.[3] In the following, we will

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[3] One might argue that the contribution to washout from $\Delta L=2$-scattering processes involving the heavy $\mathrm{SU}(2)_{\mathrm{L}}$-triplet Δ can be treated analagously to the contribution from the heavy right-handed neutrinos $\nu_{\mathrm{R2}}, \nu_{\mathrm{R3}}$. For $M_\Delta, M_{\mathrm{R2}}, M_{\mathrm{R3}} \gg M_{\mathrm{R1}}$ these heavy fields can be effectively integrated out, contributing via the same effective dimension 5 neutrino mass operator. The $\Delta L=2$-scattering processes can then be neglected for $M_{\mathrm{R1}} \ll 10^{14}\,\mathrm{GeV}\,(0.05\,\mathrm{eV}\,/\overline{m})^2$ [40], with $\overline{m}^2 := m_1^2 + m_2^2 + m_3^2$, where this is also valid in the type II see-saw case.

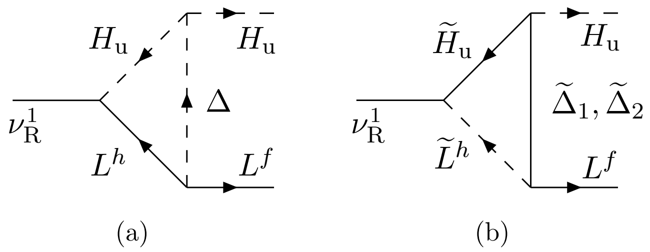


Figure 1: Loop diagrams in the MSSM involving virtual $\mathrm{SU}(2)_\mathrm{L}$-triplets, which contribute to the decay $\nu_\mathrm{R}^1 \to L_a^f H_{\mathrm{u}b}$ in the type II see-saw mechanism. Δ in figure 1(a) is the $\mathrm{SU}(2)_\mathrm{L}$-triplet Higgs coupling to the lepton doublets and $\widetilde{\Delta}_1$ and $\widetilde{\Delta}_2$ in figure 1(b) are the mass eigenstates corresponding to the superpartners of the $\mathrm{SU}(2)_\mathrm{L}$-triplet scalar fields Δ and $\bar{\Delta}$ (see e.g. [12] for details).

assume that to a good approximation η still depends only on $\widetilde{m}_1$ [8], defined in eq. (2.11), in the same way as in the type I see-saw mechanism. For our estimates, we will use the results for $\eta(\widetilde{m}_1)$ of [6]. In the scenario under consideration,

$$\widetilde{m}_1 = m_3^\mathrm{I} \, , \tag{3.11}$$

which means it decreases if the neutrino mass scale is lifted via $m_\mathrm{LL}^\mathrm{II} = m^\mathrm{II} \mathbb{1}$ (cf. eqs. (3.4) and (3.5)). Quantitatively, if we assume a neutrino mass scale $m^\mathrm{II} = 0.2\,\mathrm{eV}$, we see from eq. (3.5) that m_3^I reduces from about 0.05 eV to 0.006 eV, leading to an increase of η from about 0.003 to 0.04. Note that for $\widetilde{m}_1 = 0.006\,\mathrm{eV}$, η is still nearly independent of the initial population of right-handed neutrinos (see e.g. [6]).

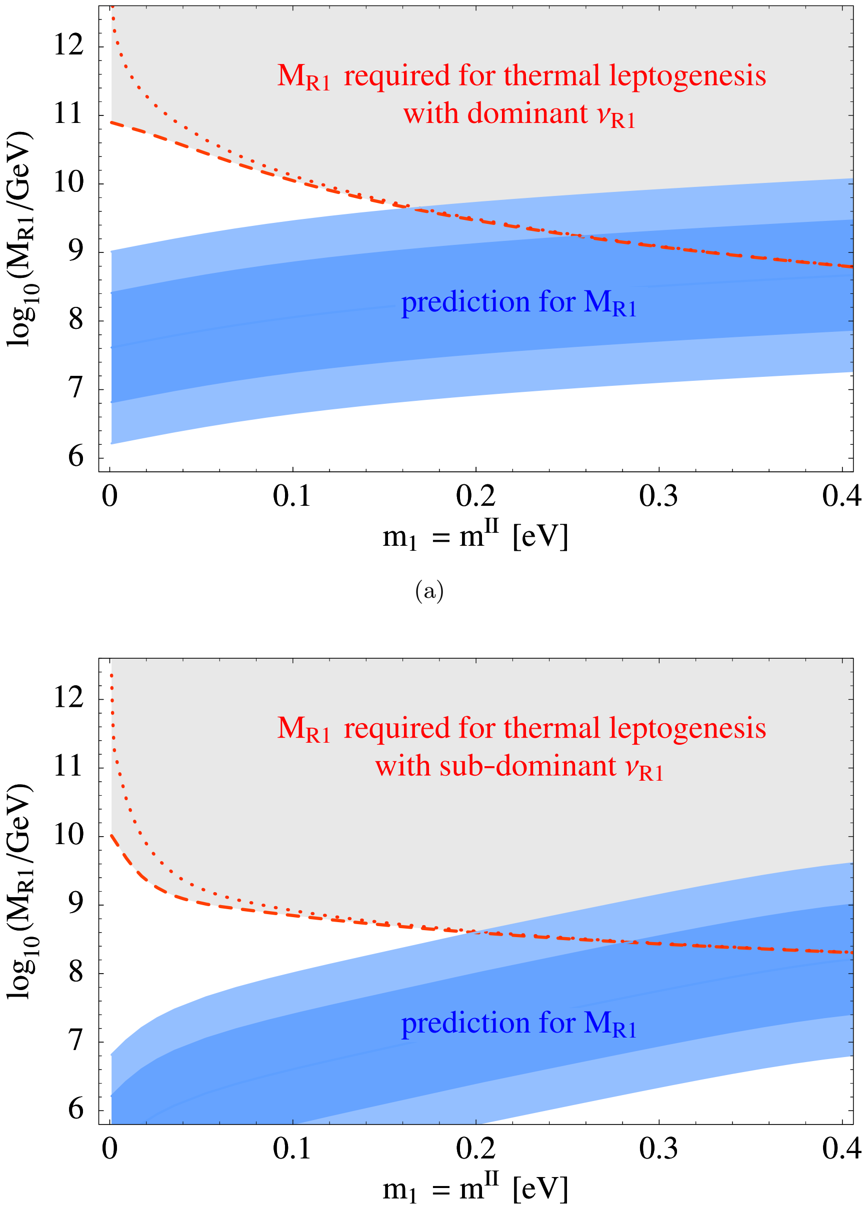
Using eqs. (2.7), (2.12), (3.4) and (3.9), the lower bound on the mass of the right-handed neutrino from the requirement of successful thermal leptogenesis can be calculated from

$$M_\mathrm{R1} \geq \frac{8\pi v_\mathrm{u}^2}{3} \, \frac{n_\mathrm{B}/n_\gamma}{1.04 \cdot 10^{-2} \, [\sin(2\delta_1 - \delta_\Delta) \, m^\mathrm{II} + m_3^\mathrm{I}]} \, \frac{1}{\eta} \, , \tag{3.12}$$

which now depends on m^II. Note that η in eq. (3.12) is calculated with $\widetilde{m}_1 = m_3^\mathrm{I}$. The lower bound on M_R1 thus decreases with increasing neutrino mass due to the explicit factor m^II in the denominator (from the decay asymmetry) and due to an increase in η.

3.3 Numerical results

We have seen that adding a type II contribution proportional to the unit matrix, leads to an increase in the prediction for M_R1 in the considered class of unified flavour models and in addition to a decrease of the lower bound on M_R1 from the requirement of successful thermal leptogenesis. Quantitatively, this is shown in figure 2(a) for a leptogenesis phase chosen to be $\delta_\mathrm{cosm} = 2\delta_1 - \delta_\Delta = 135°$. In addition, we have set the type I contribution ϵ_1^I to the decay asymmetry to its maximal value proportional to m_3^I, such that in the $m^\mathrm{II} = 0$ limit we obtain the type I bound. We have also used the same range for $e\epsilon^3$ as in the discussion of the type I see-saw models. RG effects [41] are included for $\tan\beta = 10$ as an example, using the software packages REAP/MPT introduced in [42]. We find that in unified theories where the lightest right-handed neutrino dominates the see-saw mechanism, thermal leptogenesis is possible if m^II is larger than about 0.15 eV.



(a)

(b)

Figure 2: Estimates for the mass of the lightest right-handed neutrino M_{R1} in unified theories with type II see-saw, compared to the lower bounds from successful thermal leptogenesis (dashed line) with $M_{\mathrm{R1}} \ll M_{\mathrm{R2}}, M_{\mathrm{R3}}, M_{\Delta}$. Figure 2(a) shows the results for classes of unified models where the lightest right-handed neutrino dominates the type I see-saw contribution and figure 2(b) shows the results where it is sub-dominant. The dotted lines are the leptogenesis bounds on M_{R1} with $\varepsilon_1^{\mathrm{I}}$ set to zero. Note that in the type I limit where the neutrino mass scale $m^{\mathrm{II}} = 0$ is zero, the leptogenesis bounds are more stringent than the general bound $\sim 10^9$ GeV due to larger washout and clearly in conflict with the predictions for M_{R1}. However, the bounds decrease with increasing neutrino mass scale and in addition the predictions for M_{R1} increase, allowing for consistent thermal leptogenesis if neutrino masses are larger than about 0.15 eV.

4. Leptogenesis in unified theories with a sub-dominant lightest RH neutrino

In this section we relax the assumption that the lightest right-handed neutrino dominates the see-saw mechanism, and briefly discuss leptogenesis and right-handed neutrino masses in unified theories where the lightest right-handed neutrino is sub-dominant within the type I see-saw contribution. To be precise we shall assume in this section that the lightest right-handed neutrino is mainly responsible for the type I contribution to the solar neutrino mass, while the next-to-lightest right-handed neutrino is mainly responsible for the type I contribution to the atmospheric neutrino mass. This is sometimes referred to as intermediate sequential dominance (ISD) [23]. For the neutrino Yukawa matrix, we assume the form

$$Y_\nu \sim \begin{pmatrix} a\epsilon^4 e^{\mathrm{i}\delta_1} & * & * \\ b\epsilon^4 e^{\mathrm{i}\delta_1} & e\epsilon^2 e^{\mathrm{i}\delta_2} & * \\ c\epsilon^4 e^{\mathrm{i}\delta_1} & f\epsilon^2 e^{\mathrm{i}\delta_2} & \mathcal{O}(1) \end{pmatrix}, \tag{4.1}$$

where $\epsilon_\nu = \epsilon_\mathrm{u} \approx 0.05$ and where the entries marked with a star are much smaller than the other entries in the corresponding column of Y_ν, where the RH neutrino associated with the second column dominates the see-saw mechanism, and the first column gives the leading sub-dominant contributions. An important feature is that Y_ν is linked to the quark Yukawa matrix Y_u, so that $(Y_\nu)_{11} = a\epsilon^4$ is related to the up-quark Yukawa coupling which can be estimated as $y_\mathrm{u} \approx (Y_\mathrm{u})_{11} \approx 4.7 \cdot 10^{-6}$ at the GUT scale (see e.g. [43]). We will use the range $a\epsilon^4 \in [\frac{1}{5}, 5] \cdot 4.7 \cdot 10^{-6}$ in our analysis. The mass of the lightest right-handed neutrino is then given by

$$M_{\mathrm{R}1} = \frac{(a\epsilon^4)^2 v_\mathrm{u}^2}{(s_{12}^\nu)^2 m_2^\mathrm{I}} \quad \text{where} \quad m_2^\mathrm{I} \sim \frac{\Delta m^2_\mathrm{sol}}{-2m^\mathrm{II}\cos(2\delta_1 - \delta_\Delta)}\,, \tag{4.2}$$

in the limit of large m^II, analogous to eqs. (3.4) and (3.5). Δm^2_sol is defined in the usual way as $m_2^2 - m_1^2$. We see that m_2^I decreases with increasing neutrino mass scale, even faster than m_3^I, and thus $M_{\mathrm{R}1}$ increases significantly. In the type I limit where $m_2^\mathrm{I} = \sqrt{\Delta m^2_\mathrm{sol}}$, $M_{\mathrm{R}1}$ is predicted to be in the range

$$M_{\mathrm{R}1} \sim 2 \cdot 10^4 \text{ GeV} \cdots 7 \cdot 10^6 \text{ GeV}\,, \tag{4.3}$$

clearly incompatible with requirements on $M_{\mathrm{R}1}$ from thermal leptogenesis. For $\widetilde{m}_1$ we find

$$\widetilde{m}_1 \geq m_2^\mathrm{I} \tag{4.4}$$

and the lower bound on the mass of the right-handed neutrino from the requirement of successful thermal leptogenesis can be calculated from

$$M_{\mathrm{R}1} \geq \frac{8\pi v_\mathrm{u}^2}{3} \frac{n_\mathrm{B}/n_\gamma}{1.04 \cdot 10^{-2}\,[\sin(2\delta_1 - \delta_\Delta)\, m^\mathrm{II} + m_3^\mathrm{I})]} \frac{1}{\eta}\,. \tag{4.5}$$

Note that in eq. (4.5), η is calculated with $\widetilde{m}_1 = m_2^\mathrm{I}$. In the type I limit, we obtain a lower bound for $M_{\mathrm{R}1}$ of about $10^{10}\,\mathrm{GeV}$. This bound decreases with increasing neutrino mass scale m^{II} since washout can be smaller for lower m^{I}_2 and since the decay asymmetry increases with m^{II}.

The prediction for the mass of the lightest right-handed neutrino is compared to the numerical results for the lower bound from successful thermal leptogenesis in figure 2(b). We find that for the example with $\tan\beta = 10$, a neutrino mass scale larger than about 0.2 eV is required for consistent thermal leptogenesis, assuming zero initial population of right-handed neutrinos. We note that for sub-dominant $\nu_{\mathrm{R}1}$ and m^{II} larger than about 0.03 eV, $\widetilde{m}_1$ can be smaller than $\approx 10^{-3}$ and the efficiency factor η can depend on initial conditions. On the contrary, with the lightest right-handed neutrino being dominant in the see-saw mechanism we have found that for masses up to about 0.2 eV, thermal leptogenesis is still in the strong washout regime and the produced baryon asymmetry is nearly independent of initial conditions. Furthermore, with quasi-degenerate neutrino masses RG running of the neutrino parameters between low energy and $M_{\mathrm{R}1}$ has to be taken into account carefully, in particular for the mixing angle θ_{12} entering eq. (4.2) and for the solar mass squared difference.[4] Due to RG effects, the required value of m^{II} may vary for different choices of $\tan\beta$, however this does not change the general result that a non-zero type II contribution is required.

5. Summary and conclusions

As pointed out by many authors, in some classes of unified theories where the neutrino Yukawa matrix is linked to the up-quark Yukawa matrix the prediction for the mass of the lightest right-handed neutrino is in conflict with the lower bound from the requirement of successful thermal leptogenesis. In this study, we have investigated how lifting the absolute neutrino mass scale by adding a type II see-saw contribution proportional to the unit matrix can resolve this potential problem. We found that in these classes of type II see-saw models, lifting the neutrino mass scale increases the predictions for the masses of the right-handed neutrinos while the decay asymmetry for leptogenesis is enhanced and washout effects are reduced, thereby relaxing the lower bound on the mass of the lightest right-handed neutrino from thermal leptogenesis. A type II see-saw contribution proportional to the unit matrix can be realized using for instance SO(3) family symmetry or discrete symmetries. It provides a natural way of transforming a type I see-saw model for hierarchical neutrino masses into a type II see-saw model for quasi-degenerate neutrinos.

We have mainly focussed on classes of unified theories where the lightest right-handed neutrino dominates the type I see-saw mechanism, and where sequential dominance provides a natural mechanism for giving a neutrino mass hierarchy and bi-large neutrino mixing angles in the presence of small charged fermion mixing angles. We have shown that in this type I see-saw scenario for hierarchical neutrino masses, the prediction for the mass of the lightest right-handed neutrino is in conflict with the lower bound from the requirement of

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[4] For quasi-degenerate neutrino masses, the running of θ_{12} is generically much stronger than the running of the other mixing angles [44], in particular with a dominant type II contribution proportional to the unit matrix, which implies small Majorana CP phases [24].

successful thermal leptogenesis. We have then discussed in detail how lifting the absolute neutrino mass scale by adding a type II see-saw contribution proportional to the unit matrix can resolve this conflict. We have found that thermal leptogenesis becomes possible with a neutrino mass scale larger than about 0.15 eV, which implies observable neutrinoless double beta decay (and possibly also a signal from direct neutrino mass searches) in the near future. For such neutrino masses, thermal leptogenesis remains in the so-called strong washout regime, where the produced baryon asymmetry is virtually independent of initial conditions.

We have also discussed classes of unified models where the second lightest right-handed neutrino dominates the type I see-saw mechanism, and the lightest provides the leading sub-dominant contribution. In such models the prediction for the mass of the lightest right-handed neutrino is also conflict with the lower bound from thermal leptogenesis, and again this conflict may be resolved by a type II see-saw up-grade similar to the previous case of a dominant lightest right-handed neutrino. However, if the heaviest right-handed neutrino dominates the see-saw mechanism, and the lightest right-handed neutrino is effectively decoupled, then there is generically no conflict between leptogenesis and unified models, but the Yukawa matrices must involve large mixings.

Acknowledgments

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